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Visual Thinking in Fractions: Exploring Elementary Students' Conceptual Understanding through Representation Strategies

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ABSTRACT

This study explores fifth-grade students' use of visual representations in their understanding of fraction operations and what their drawings show us about their underlying conceptual understanding. A new and comprehensive exploratory qualitative collective case study was carried out with 15 students from three public elementary schools in Makassar, Indonesia. Data were obtained through a visual fraction addition representation task, think-aloud problem-solving, and semi-structured student and teacher interviews. Based on a thematic analysis by interactive methods, the study discovered various strategies of representation, including area diagrams, number lines, bar models, and hybrids of the diagrams and bars. Students made good use of visual representations consistently and in numbers, especially area diagrams and combined representations and had a strong concept for fraction equivalence, common denominators, and magnitude and had a higher flexibility in transitioning between representations. Students who could not keep the unit consistent or scale proportionally consistently often resulted in critical conceptual errors, on the contrary. The results imply that visual representation is not something we can just use to decorate a diagram when reasoning about fractions, but rather it needs to form an intrinsic part of students' brains' thinking toolkit to help students interpret and apply fractions. The research thus suggests a need for targeted teacher professional

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development that is focussed on the design, interpretation, and scaffolding of visual models for fraction instruction, which allows teachers to scaffold deeper conceptual understanding and support students' overall mathematical literacy.

Keywords: Conceptual understanding, elementary mathematics, fraction operations, visual representation

INTRODUCTION

Students' understanding of fractions is one of the biggest challenges of primary mathematics education. A fraction asks children to work out several ideas at once: to see numbers as parts of a whole; to develop a sense of proportion; to interpret symbols; and to construct mental images. There is a wealth of evidence indicating that difficulties with fraction operations represent a significant barrier to math learning later (Lamon, 2012; Siegler et al., 2011) and to problem solving in upper grades (Dyson et al., 2018; Gersten et al., 2017). Moreover, a well-known "natural number bias" may prevent students from judging the magnitudes of fractions (Van Hoof et al., 2018). Such persistent misconceptions require pedagogically designed solutions that blend procedural knowledge with conceptual significance. One hopeful avenue is to incorporate visual tools such as area diagrams, number lines, or bar models that link abstract symbolic expressions with concrete images (Soni & Okamoto, 2020). Recent meta-analytic findings suggest that visual-based interventions possess at least a moderate effect on mathematics achievements, especially for fractioning tasks (Braithwaite & Siegler, 2020).

Visual thinking is not just a way to have something nice to look at, but a primary way of thinking about and working with ideas in mathematics. Visual representation (e.g., area diagrams and number lines, fraction pieces, bar models) enables children to create a mental representation of what fractions are and assists them in reasoning while solving math (Cramer et al., 2002). When quantities are visible in something tangible (that is, concrete and contextual), visualisation allows learners to perform procedures while following the rational construction of each stage of them. The National Council of Teachers of Mathematics (NCTM, 2020) considers representations as "absolutely vital" for the development of mathematical concepts and relationships. However, the pedagogical potential of visual representation is still underused in many classrooms, particularly in developing countries where instruction tends to emphasise rote memorisation over conceptual understanding.

Recent studies indicate that representational instruction yields better results in procedural and conceptual fraction tasks than peers taught exclusively in symbols (Fazio & Siegler, 2011; Van de Walle et al., 2019). Strong visual models (such as the two-dimensional number lines, the dynamic bar model and triple number lines) also reinforce students' learning (Reinhold et al., 2020; Wortha et al., 2020). For instance, Susanti et al. (2022) conducted studies in Indonesia; they confirmed that elementary students using number lines and circle diagrams in addition to and subtraction of fractions had a better understanding than students with symbolic procedures only. Context-based mathematics education that uses visual aids and interactive media has also been empirically supported as facilitating fraction sense as a matter of life (Kamberi et al., 2022; Pramudiani & Dolk, 2022). Collectively, these findings suggest the trend in favour of instruction that relies on visualisation-based meaning making. Yet, we do not yet see much empirical evidence on how students build, use and understand visual models of fractions when working with fractions in their Indonesian classrooms. Addressing this gap is the focus of the present study.

Even when visual learners are present in the classroom, they may struggle to follow rapid symbolic explanations on the board (Ramsden, 1992). Several studies indicate that teachers often possess only limited knowledge of how to integrate visual representations effectively. For example, Putra et al. (2020) reported that many teachers view pictures and models merely as illustrations rather than mathematical tools that can support reasoning. A large-scale survey of 350 primary teachers likewise found that images were typically used as decorative elements rather than as resources for mathematical discussion (Gencturk & Doleck, 2021). Other research stresses that visual models serve specific purposes: number lines are well suited for ordering fractions and comparing magnitudes, whereas circle diagrams may better communicate part-whole relationships (Wilkie et al., 2022). At the same time, many teachers report low confidence in using more complex visual tools such as number lines for fraction instruction (Putra et al., 2020). These findings underline the need to strengthen teachers' representational competence through professional development that links theoretical understanding of models with practical classroom use.

International curriculum analyses show that visual representations play a central role in mathematics textbooks and teaching in many high-performing education systems. Charalambous et al. (2022) reported that countries such as Singapore, Finland, and the United States routinely include diverse visual models for fractions and encourage students to transition from concrete images to more abstract representations. Finnish textbooks, for instance, integrate exploratory activities with visual aids rather than relying solely on static pictures. Curriculum documents in Canada focus on the unit fractions and number lines used for representational flexibility (Bruce et al., 2023), and a six-year mixed-methods study involving more than 2,000 students and 86 teachers observed large improvements when instruction was explicitly designed to bridge concrete experiences with abstraction (Bruce et al., 2022). The implications for mathematics teaching in practice from these international trends are clear: Visual representation is not a bolt-on, but an essential aspect of effective mathematics learning. They also underscore the need to think about how visual strategies are used to teach in Indonesia's specific educational setting.

The importance of visual representation is also supported by cognitive and sociocultural theories of learning. Paivio's (1990) dual-coding theory proposes that verbal information and visual imagery are processed in parallel systems, leading to stronger memory and deeper comprehension. In fraction learning, for example, students who can connect the symbolic expression ¾ with an area model and the verbal description "three parts out of four equal parts" are likely to form richer mental representations than those who encounter only symbols. Furthermore, recent neuroimaging studies have shown that number and spatial representations in mathematical tasks activate brain regions related to quantitative reasoning, and that continued exposure to visual models over time can facilitate neurofunctional changes related to mathematical thinking (De Smedt et al., 2011; Hwang et al., 2019; Lenz & Wittmann, 2020; Tian et al., 2021; Wortha et al., 2020). Visual models can help democratise access to meaning from a sociocultural and critical-education perspective by enabling diverse students to "see" mathematical structures before dealing with more abstract forms (Prediger et al., 2018; Karamarkovich & Rutherford, 2019). At the same time, different models of visual representation offer different kinds of understanding: area diagrams are used successfully to model part-whole relations, but may prove more challenging for operations that require unlike denominators, whereas number lines provide visual representation of order and magnitude but can be misleading for learners who don't yet view fractions as points on a continuum (Barbieri et al., 2020; Clarke & Roche, 2018). As a result, teachers and curriculum designers will need to assess which visual models are appropriate for the purposes if representations are to function as tools for deep understanding rather than superficial decoration.

Building on this theoretical and empirical background, the present research is guided by the view that visual representations can serve not only as instructional aids but also as analytic windows into students' conceptual understanding of fractions. Rather than focusing solely on the effectiveness of particular interventions, this study examines how students themselves construct and use visual models when solving fraction problems, and what their drawings reveal about their reasoning with equivalence, common denominators, and magnitude. More specifically, we adopt a collective case study design to investigate fifth-grade students' representational strategies with number lines, area diagrams, bar models, and combinations of these. From a practical perspective, the study aims (1) to describe the kinds of visual images students produce when calculating fractions and (2) to identify the conceptual understandings and misconceptions embodied in these representations, with reference to teachers' views of classroom practice. By examining these dimensions, the research seeks to contribute to curriculum design and instructional approaches that prioritise conceptual understanding over rote procedures and to support Indonesian students' mathematical literacy. Consistent with Goldin's (2002) notion of representational flexibility, we emphasise how students move between images and symbols when working with fractions. Accordingly, this study addresses the following research questions: (RQ1) How do fifth-grade students represent fraction operations using visual models such as area diagrams, number lines, and bar models? (RQ2) What do these visual representations reveal about students' conceptual understanding and misconceptions related to fraction operations? (RQ3) How can the patterns in students' visual representations inform teachers' use of visual models in classroom instruction?

LITERATURE REVIEW

Theoretical Foundations of Visual Representation

Doing mathematics requires students to both understand and represent mathematical ideas. Theoretical work in mathematics teaching asserts that representations are not marginal but crucial to understanding. According to Duval's (1999) semiotic representation theory, understanding mathematics is dependent on the processing of information as it is directed across various semiotic systems, those systems being symbolic, verbal, graphical, and figural. In a similar vein, Goldin (2002) highlights internal and external representation systems that mediate cognitive processes associated with solving problems. Ainsworth's (2006) Design, Functions, Tasks (DeFT) framework outlines how multiple representations function in learning: a complementary function, where representations highlight different aspects of a concept (a process); a constraining function, where specific representations minimise potential misreading; and a constructing function, where learners create deeper structures by coordinating representations. In combination, these viewpoints indicate that students learn best when they can easily adapt symbols, pictures, and verbal descriptions, using representations to articulate and develop their mathematical thinking.

Visual Representations and Conceptual Understanding of Fractions

There is empirical evidence that visual modalities like area diagrams, number lines, fraction bars, and circle models can enhance students' reasoning regarding part-whole relationships and proportionality (Cramer et al., 2002; Soni & Okamoto, 2020). Meta-analyses report medium-to-large effects of visual-based instruction on mathematics learning outcomes, particularly in the domain of fractions (Braithwaite & Siegler, 2020). More fine-grained studies suggest that a dynamic bar model can enhance both procedural and conceptual fluency

(Reinhold et al., 2020), while the engagement with visual tasks activates regions of the brain that are directly responsible for reasoning and memory (Wortha et al., 2020). Neuroimaging evidence also indicates that long-term visual modelling can build neurofunctional plasticity, so students can reason about the magnitudes of fractions (Tian et al., 2021). Taken together, these results situate visualisation as a fundamental cognitive mechanism for fraction learning rather than as a surface or optional teaching tool.

Challenges in the Use of Visual Representations

Even though there is a wealth of evidence that supporting the effectivness of visual representations, significant challenges remain in their classroom use. Research suggests that many teachers are not confident or knowledgeable about how to effectively integrate visual tools, and diagrams often do so with little pedagogical focus on them as aids of reasoning. Instead, many treat the diagrams as decorative pictures (Putra et al., 2020; Gencturk & Doleck, 2021). Wilkie et al. (2022) point out that model choice needs to be in relation to the learning intentions; number lines are particularly appropriate for modelling fractions, and area diagrams are beneficial for part-whole relationships. In addition, if visual models are not consistent in their scale or are not linked to symbolic notation, they can block comprehension and result in systematic conceptual misunderstandings (Barbieri et al., 2020; Clarke & Roche, 2018). These findings highlight the need for targeted teacher professional development that builds representational competence allowing teachers to select, format, and interpret visual models in a manner that actively supports students' understanding of concepts.

International and Indonesian Contexts

Visual representation is an important element of many mathematics curricula around the world. International analyses demonstrates that high-performing systems such as Singapore and Finland systematically employ visual models to assist students in moving from concrete situations to more abstract ideas. In the Indonesian context, studies by Susanti et al. (2003) and Kamberi et al. (2004) report that activities involving number lines and circle diagrams can help students better understand fractions. However, many barriers remain in their implementation, such as limited resources in the classroom and a strong bias towards teaching students step by step. Pramudiani and Dolk (2005) argue that connecting Realistic Mathematics Education (RME) with visual strategies can help bridge the gap between informal, context-based reasoning and formal symbolic representations. This raises a specific need for detailed classroom evidence regarding the process of constructing and utilising visual representations among Indonesian learners using fractions, which is what this study endeavours to address.

METHODOLOGY

Research Design

This exploratory qualitative study employed a collective case study design to explore how elementary school students build and manipulate pictures to understand fraction operations. The tool was selected with the aim of providing a contextualised understanding of students' mathematical thoughts with the use of visual representations (number lines, area diagrams, and bar models). A collective case study design allowed researchers to investigate various single cases (individual students from different schools) to identify trends and differences in representational strategies (Creswell & Poth, 2018). In such a framework, each student was conceived of as "a bounded case, situated inside a particular school context and thus enabled specific investigation of visual productions, but encouraged comparison among cases. Given the focus on in-depth understanding, a small sample of 15 students was considered appropriate to address the research questions, prioritising rich description over statistical generalisation. The research emphasised depth of understanding rather than statistical generalisation, focusing on how students constructed conceptual meaning through visualisation (Goldin, 2002).

Participant Selection

The participants consisted of 15 fifth-grade students from three elementary schools in Makassar, South Sulawesi, Indonesia: five from SD Inpres Andi Tonro, five from SDN Manuruki, and five from SDN Minasaupa. Participants were selected through purposive sampling based on diagnostic test results and teacher recommendations, with the following criteria: (1) prior exposure to fraction operations, (2) representation of diverse mathematical ability levels (low, medium, high), and (3) willingness to engage in visual exploration activities. Selecting students from three different schools within the same city provided variation in classroom environments while maintaining a shared curricular and cultural context. In addition, three mathematics teachers participated as triangulative informants to enrich and validate data interpretation. This purposive sampling approach aligns with the qualitative paradigm, which prioritises information richness over population representativeness (Moleong, 2021).

Data Collection

Data were gathered using three complementary techniques: Visual Representation Task (VRT), a written task requiring students to solve fraction problems visually. For instance, they were asked: "What is the result of 2/3 + 1/4? Solve it using a picture and explain your reasoning! Think-Aloud Protocol (TAP), students verbalised their reasoning while performing the tasks. Audio and video recordings captured their thought processes and strategies (Ericsson & Simon, 1993). Semi-Structured Interviews, guided by Janvier's (1987) representational framework, these interviews explored students' reasoning for representation choices, their challenges, and the conceptual meanings constructed.

Interviews with teachers were also conducted to gather pedagogical insights about classroom implementation of visual representations. These three techniques provided complementary perspectives on students' visual strategies and underlying reasoning, thereby directly addressing the research questions.

Data Analysis

Data was analysed using interactive thematic analysis (Miles et al., 2014). First, all visual representations, think-aloud transcripts, and interview recordings were transcribed and organised. Second, the researchers conducted open coding to identify initial categories related to types of representations, unit consistency, equivalence reasoning, and representational flexibility. Third, these codes were grouped into broader themes through constant comparison within and across cases, with particular attention to similarities and differences among students and schools. Coding decisions were discussed among the research team, and discrepancies were resolved through peer debriefing to enhance credibility. The resulting themes were iteratively refined through repeated reading of the data and were then used to construct a cross-case narrative that addressed the research questions.

Trustworthiness

Credibility and dependability were established through methodological triangulation, data-source triangulation, member checking, and peer debriefing Audit trails were maintained throughout the process to ensure transparency and confirmability.

Ethical Considerations

Ethical clearance was acquired before data collection took place at Institute for Research Development, Community Service (LP3M) Muhammadiyah University Makassar. School principals, teachers and parents of participant students gave written informed consent. All participants received assurance from both their teacher and the researcher that participation is confidential, anonymous, and voluntary.

Research Procedure

The study was implemented in five stages: (1) instrument preparation, (2) participant selection, (3) administration of the VRT and TAP, (4) in-depth interviews, and (5) data analysis and reporting.

RESULTS

This section presents the results based on data collected from 15 fifth-grade students and three mathematics teachers through a Visual Representation Task (VRT), Think-Aloud Protocol (TAP), and semi-structured interviews. The findings are organised by themes to provide an overview of how students used visual strategies, the typical features of their representations, and indications of their conceptual understanding of fraction operations. Where relevant, figures and tables are included to illustrate key patterns in the data.

Types of Visual Representations Used by Students

Analysis of the VRT showed that students employed a variety of representations when solving fraction addition problems, particularly for the question $\frac{2}{3}+\frac{1}{4}$. Among the 15 students, five used number lines (S1-S5), four used area diagrams such as circles or rectangles (S6, S7, S9, S10), and three used bar models (S12, S14, S15). Three students (S8, S11, S13) combined more than one type of visual model, most often number lines and area diagrams. Figure 1 presents the distribution of representation types. This variation suggests that students did not rely on a single dominant model but chose representations that were familiar or accessible to them, with some students drawing more than one model within the same task.

During interviews, students explained their preferences for particular models. S1 noted, *"I often see number lines in the textbook, so I used that because I thought it was the correct way,"* indicating reliance on familiar textbook formats. In contrast, S13 explained, *"I used boxes to help me see how many parts each fraction has, but the line helped me add them better,"* describing a deliberate use of both area diagrams and a number line within the same solution.

Number Line Representation: Accuracy and Challenges

Number lines, although commonly chosen, reveal both accurate and inaccurate reasoning. Students such as S2 and S5 partitioned the number line correctly and plotted the two fractions in appropriate positions. However, others, including S4, made notable errors. Figure 2 shows S4's work for $\frac{2}{3}+\frac{1}{4}$. S4 divided the interval from 0 to 1 into three equal parts to represent $\frac{2}{3}$, but did not maintain a consistent unit when adding $\frac{1}{4}$. Instead of partitioning the next segment into four equal parts, S4 marked an additional point without adjusting the scale, leading to an incorrect location for the final result.

In the think-aloud session, S4 stated: *"I started from zero this is one I divided it into three parts two steps, that's two-thirds then from there I added one-fourth uh, I'm confused about how many parts this should be?"* When asked in the interview why the second part of the number line was not divided into four equal sections, S4 explained, *"I thought I could just keep going, because one-fourth is small. But I didn't know whether the sizes had to be the same or not."* These comments show that S4 recognised the need to mark both fractions on the line but was unsure how to maintain equal partitioning when adding 1/4.

Area Diagram Representations: Supporting Conceptual Reasoning

Area diagrams appeared particularly helpful for some students in solving the fraction of addition problem. Student S7, for example, produced a representation that showed attention to equivalence and common denominators. As shown in Figure 3, S7 drew two identical rectangles and divided each into 12 equal parts, corresponding to

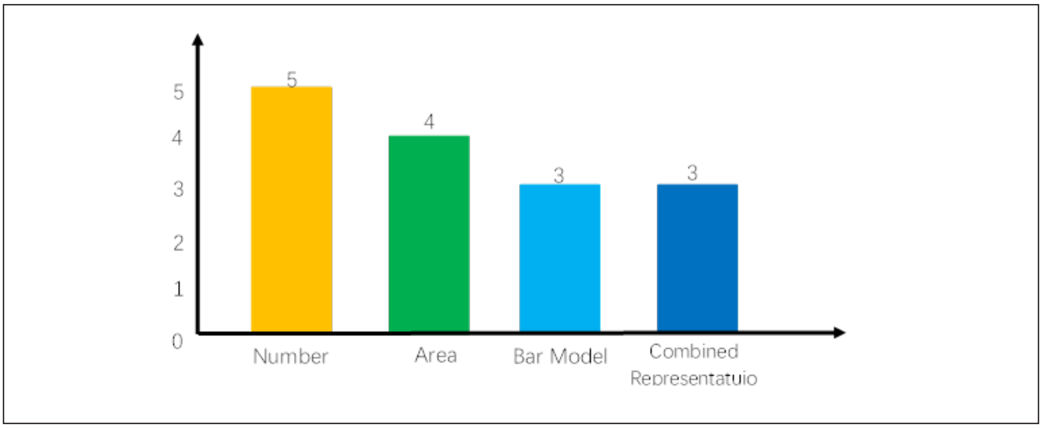

Figure 1. Distribution of visual representation types used by students

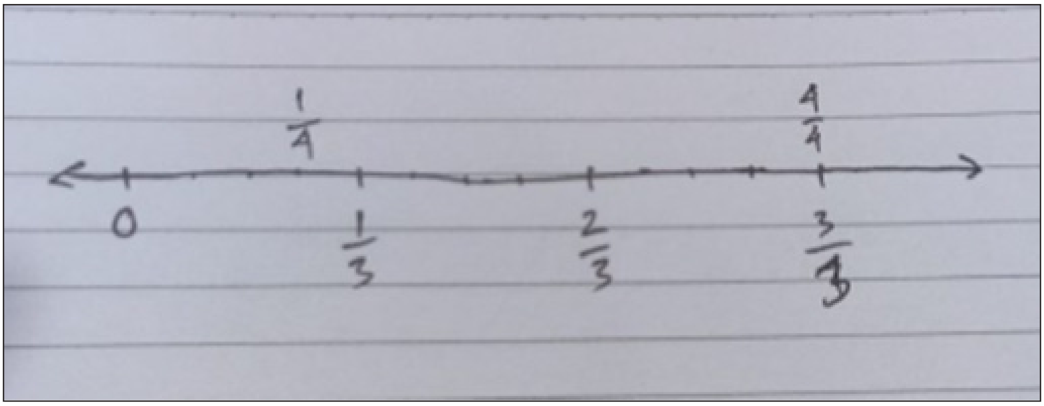

Figure 2. Results of the solution using S4 visualisation

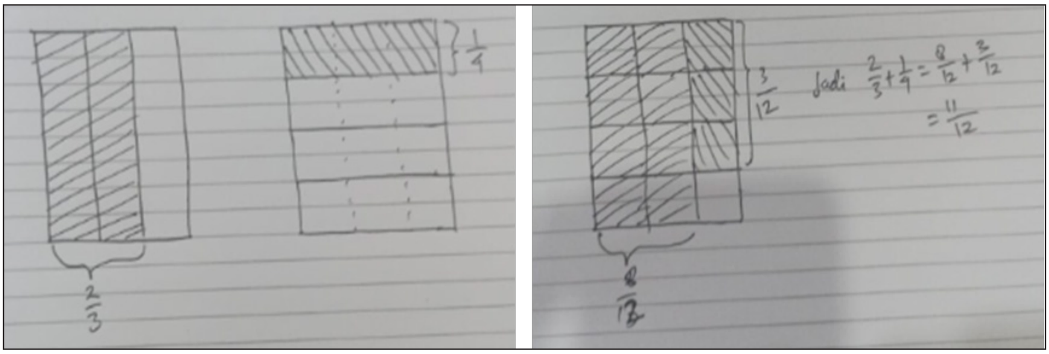

Figure 3. Results of the solution using S7 visualisation

the least common multiple of 3 and 4. The student shaded 8 parts to represent $\frac{2}{3}$ and 3 parts to represent $\frac{1}{4}$, and recorded the sum as $\frac{11}{12}$.

During the TAP, S7 explained: *"I made two equal boxes, then divided them into 12 because that works for three and four. Two-thirds means eight, because three parts of twelve is four, times two make eight. One-fourth is three. So, the total is eleven out of twelve."* This explanation shows that S7 coordinated the symbolic fractions with their equivalent twelfths in the diagram. In the interview, the teacher commented, *"This student really likes to draw. They often ask for graph paper or make their own shapes to understand fractions."* Together, the drawing and explanation indicate that S7 used the area model to organise the conversion to a common denominator and to determine the final sum.

Combined Representations: Indicators of Thinking Flexibility

Some students demonstrated the ability to transition between different representation types. Student S13, for instance, began with an area diagram to determine common denominators and then used a number of lines to finalise the solution. Figure 4 depicts S13's work, which includes two visual phases: the first showing two rectangles partitioned into twelfths and labelled with fractional values; the second, a number line equally divided into 12 segments with the final point marked at $\frac{11}{12}$. An area diagram showing the conversion of $\frac{2}{3}$ and $\frac{1}{4}$ into $\frac{8}{12}$ and $\frac{3}{12}$. A number line divided into 12 segments from 0 to 1, with a final marker at $\frac{11}{12}$.

During the think-aloud, S13 explained: *"I drew two boxes first so I could figure out the parts, then I thought 12 would work. After that, I used the number line because it looked clearer."* This case illustrates representational flexibility with the ability to coordinate and transition between visual models to enhance understanding. S13's process reflects a growing mathematical maturity, emphasising conceptual over procedural thinking. When asked why they switched to a number line, S13 noted, *"Because I was already sure about the answer and just wanted to see where it lands."* Such transitions are part of the interrepresentational transformation framework that emphasises representational competence as the foundation of mathematical reasoning.

Patterns and Variations Across Representational Strategies

Table 1 summarises the types of representations used, the associated conceptual accuracy, and key observations across all participants. Students who made use of area diagrams and combined models more frequently gave answers that followed common denominator reasoning, whereas those relying solely on number lines or bar models showed more frequent errors, especially related to unit scaling and conversion of denominators.

Interviews with teachers offered further perspective on these patterns. One teacher noted, *"Most students can draw the models,*

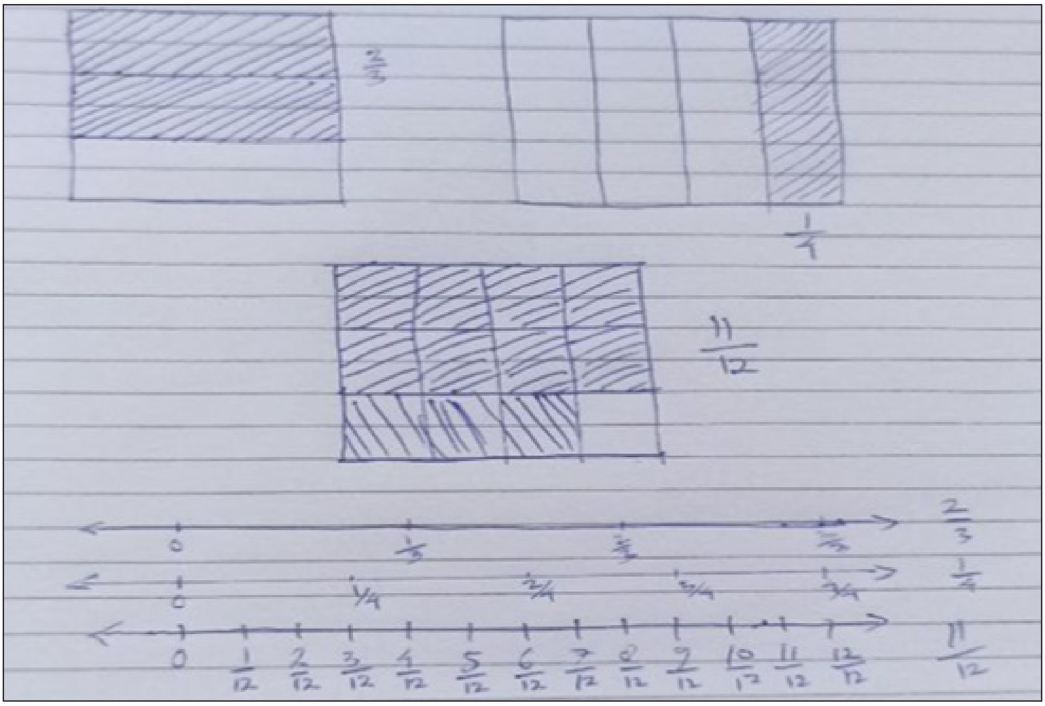

Figure 4. Results of the solution using S13 visualisation

Table 1
Summary of students' approaches in solving fraction problems

Student(s)	Approach Used	Representation	Conceptual Success
S4, S5	Number Line	Inconsistent unit size	Partial
S7	Area Diagram	Correct conversion	High
S13	Area + Number Line	Model transformation	High
S12, S14, S15	Bar Model	Missed the common denominator	Partial
S8, S11	Combined	Accurate integration	High
S1, S2, S3	Number Line	Mixed success	Mixed
S6, S9, S10	Area Diagram	Accurate and proportional	High

but they don't always know what they mean mathematically." Another commented, *"We need more training to not just show students how to draw, but how to think with the pictures."* These comments demonstrate that teachers noticed a disconnect between students' capacity to produce visual models and their ability to utilise the visual models to communicate mathematical ideas.

Misconceptions and Pedagogical Implications

Several students displayed misconceptions even when they used visual models.

For example, S12 used a bar model by drawing separate bars for $\frac{2}{3}$ and $\frac{1}{4}$ and then simply counting all the segments together without adjusting to a common denominator. When asked about this solution, S12 stated, *"I just counted the pieces. It looks like ten parts total.* In interviews, teachers acknowledged similar issues. One teacher noted, *We were trained how to use these models but not how to connect them with concepts such as common denominators and equivalent fractions.* These findings show that some students treated the segments in their drawings as unrelated pieces rather than as parts of a single whole, and that teachers themselves felt a need for further support in helping students interpret visual models conceptually.

Summary of Key Findings

In general, the results suggest that students who presented the visual representations in a consistent and proportional manner had a better understanding of fraction operations. Area diagrams were the best predictors of correct reasoning relating to common denominators and equivalent fractions, whereas number lines were supportive for showing fraction magnitudes but more susceptible to errors in unit consistency. Students who combined different models, such as area diagrams and number lines, tended to produce more complete and coherent solutions. These key patterns from the results provide the basis for the interpretation and implications discussed in the following section.

DISCUSSION

This study discovered that students' understanding of fraction operations was primarily determined by the manner of use of pictures. The students who were able to use an area diagram/bar model which followed a uniform partitioning presented a better understanding of common denominator reasoning and the relations of part and whole. These themes echo the prior findings (Cramer et al., 2002) that area-based models are good for connecting our symbolic and visual thinking. When students employed figures such as these, they were able to obtain not only correct answers but to explain their reasoning, indicating that visualisation facilitated a reliable cognitive hierarchy of fraction concepts. This argument is in line with Duval's (1999) semiotic theory of representation and Ainsworth's (2006) DeFT framework, which underscores how multiple representations facilitate gaining depth in understanding. It is also consistent with general evidence support that visual-based interventions in fraction learning provide medium-to-large effect sizes (Braithwaite & Siegler, 2020; Soni & Okamoto, 2020) and that number-line techniques can strengthen students' sense of fraction magnitude (Teoh et al., 2021).

The results also highlight the importance of students' ability to move between different representations, for example, from area diagrams to number lines, as an indicator of representational competence. Cases such as S13, who coordinated area models and number lines within a single solution, illustrate what

Goldin (2002) describes as flexibility of representation, a key prerequisite for robust mathematical understanding. This flexibility echoes Janvier's (1987) notion of inter-representational transformation and Duval's (1999) account of coordinating several sign systems in mathematical cognition. External evidence points in the same direction: Wilkie et al. (2022) showed that tasks involving multiple visual representations of the same fraction concept can deepen understanding, while Teoh et al. (2021) reported that students who worked with number lines and intervals showed improved transitions between representations. Taken together, these findings underline that, even in the early stages of fraction learning, fluency in multiple representational forms is not optional but central to developing more abstract reasoning.

At the same time, recurring errors in students' drawings, such as inconsistent scales, mismatched units, and failure to coordinate models with symbolic notation, show that the presence of a visual representation does not guarantee conceptual understanding. The findings suggest that explicit instruction is needed to help students interpret, construct, and check their representations. This is consistent with work by Putra et al. (2020), who reported that Indonesian teachers recognise the potential of visual tools but often lack the pedagogical content knowledge to use them effectively. International studies echo these concerns: Clarke and Roche (2018) found that students misinterpret number lines when unit partitioning is not made explicit, and Barbieri et al. (2020) showed that natural number bias continues to influence fraction magnitude judgments even in the presence of visual aids. Accordingly, teacher professional development should focus not only on introducing visual tools but also on cultivating representational thinking, helping teachers and students to connect different visual models and to symbolic expressions.

Overall, this research supports the view that visual representations are not "pretty pictures but pragmatic tools for cognition. In classrooms that aim for idea-rich teaching and learning, teacher explanations, textbook examples, and student tasks all need to position visual models as integral to mathematical reasoning. This perspective is consistent with the NCTM (2020) standards, which argue that deep understanding rests on connections among visual representations, symbolic expressions, and contextual problem situations. For teachers, curriculum developers, and policymakers, this implies a responsibility to design materials and professional learning opportunities that link visual models explicitly to conceptual goals. Future instructional resources should therefore include guidance on how to design, sequence, and connect visual tasks so that visual thinking contributes directly to numeracy and critical reasoning. Recent meta-analytic work (Schoenherr et al., 2024) further suggests that when external representations are integrated thoughtfully, they can support both memory and higher-order reasoning. The present findings reinforce this claim and point to the need for visual representation to be treated as an everyday way of thinking in fraction instruction, rather than as a peripheral or incidental feature of lessons.

CONCLUSION

This study demonstrated how visual representations are critical to students' ability to conceptualise fraction operations, and how they create, practice, and bring together area diagrams, number lines, and bar models in relation to the research questions. Students who used area diagrams, number lines, or bar models with skill and continuity displayed greater representational flexibility, a better understanding of common denominators, and stronger reasoning to explain their math. Visual deviations like inappropriate scaling or unit mismatches frequently contributed to conceptual blunders; however, underscoring the need to connect visual models explicitly to the underlying structure of fractions. These findings highlight that visualisation is far more than an ornamental enhancement; it is a cognitive activity. As a result, teachers should be trained not only to use the visual tool but also to interpret students' drawings and to help steer students to think mathematically while working with it. The study also stressed the importance of curriculum designs which encourage meaningful and systematic employment of visual models rather than mere incidental exposure. On a theoretical level, this research enriches our understanding of the role which visualisation plays in mathematical learning, particularly within an Indonesian classroom context. Future research could extend this work through longitudinal studies that trace the development of students' visual strategies over time, design-based interventions that foreground visual representations in fraction instruction, and teacher-focused studies that examine how professional development can strengthen teachers' representational competence. Appropriate visual strategies can bridge the gap between procedural fluency and conceptual depth, enabling students to develop mathematical literacy and critical thinking skills that are relevant to real-world problem solving.

CONFLICT OF INTEREST

The authors declare that no financial, professional, or personal conflict of interest could influence the conduct, analysis, or reporting of this research. All research procedures were conducted independently and objectively, and the funding received did not influence data interpretation or conclusions. Additionally, this article is an original work that has not been previously published and is not under review or consideration by any other academic journal.

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